Folding the circle in half is a text book of information.

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Abstract

This paper addresses folding the circle in half and discussing some of over one hundred different mathematical terms and functions generated in that one fold. The simplicity of process in understanding fundamentals of mathematics by folding circles and observing what is generated is unknown because we only draw pictures of circles. Examples are given about observing and exploring relationships in the circle that are appropriate for first, second, third grade level and beyond. The traditional educational 'parts-to-whole' approach can only be fully realized through the comprehensive frame of Whole-to-parts by folding the circle. Wholemovement of the circle is not only direct; it is the only context inclusive to progressively understanding parts within unity of the Whole.

Introduction

The image of the circle is static; the circle disk in space is dynamic. This means the image does not generate anything and the circle generates information that can not be anticipated from the image. Through formulized and codified static images we have developed a particular kind of understanding about relationships, transformations, movement and change, using symbolic representations of parts in limited context. The generalized truths of mathematical functions are inherent in the movement of the circle and are revealed through a sequential process of folding, starting with the first fold.

The circle is the only shape that can through movement generate all other shapes and forms while remaining Whole. All circles inherently hold information for polygons, polyhedra, and mathematical functions. The circle is both Whole and part simultaneously. Nothing else functions in this way. This is much like a stem cell that is both an individual cell and yet holds all information for all other cell functions. We now know all cells contain this potential stem-information. The circle is a spherically compression and through folding the circle spherical information is decompressed in polyhedral form.

Logic Systems

Constructing a diameter through the circle image divides it in two semi-circles. Logic says joining two half circles makes one whole circle. When folding the circle in half, simply by touching any two points on the circumference together and creasing, we find six half circles. This goes against mathematical logic, a condition of how we generally think about things. The paper circle is a disk in space and by assessing the properties we find two circle planes and one circle ring around the middle. The three circle planes divided by that one fold makes six semi-circles. The folded crease is a diameter and given an infinite number of diameters in the circle, we can conjecture there is minimum two, if not six, times the number of half circles as there are diameters. This only when starting with the Whole. An infinite number of semi-circles will never complete the Wholeness of the circle. One form of the circle is not exclusive to the other. The circle and image co-exist as different forms; both show consistency within the limitations of the individual systems.

We did not totally accept Euclid's parallel postulate because it does not hold true for a spherical surface. It works only on a flat plane. Eventually we found a hyperbolic plane that happens between the movement from a straight line to the circle, from the flat plane to a spherical surface. None of these functions are exclusive from the others. The separation has been a convenience for lack of understanding the connections within a greater context. The circle and the image are simply neighbors in a larger community where conventions get formalized into rules and laws. All local functions in all communities are held in a global context on a cosmic map of mathematics development.

To understand the nature and the dynamics of the circle a shift from the image to the actuality of the circle is necessary. Using the circle image as a symbol for nothing, zero, is consistent to the nature of images, which have no inherent value. It is necessary to get beyond the concept image in math education if there is to be movement towards a more comprehensive and progressive understanding in a larger and more meaningful context.

We define the idea of the circle as image by the tool we use to draw it. We are stuck there. Thinking the circle has a center point reveals an inconsistency between the concentric nature of the circle and using a compass to construct the image. If the circle boundary goes infinitely out, and the circle boundary goes infinitely in, then there is no center or outer limits to the circle. The circle is the center, and the only limitation is origin and relative scale. Recursive set theory suggests infinity even bigger than infinity itself. That has been similar thinking about one universe, until recently having discovered other universes beyond universe. We get caught in the words, thus limiting our observations, holding us to past ideas, and static concepts represented by symbols. Words change to keep up with experience, but the underlying logic changes little and the inconsistencies go unnoticed. Only through active experience and observation can a system of logic be changed giving way to progressively uplifting our understanding.

Whole-to-parts

This observation of six halves would be of mild interest if four halves were the only difference between the circle and the image. When over one hundred mathematical functions describing a variety of relationships between just a few parts can be observe in this one fold in half, we need to rethink our understanding about the circle. The beauty is that all parts come from a single movement of equal division into the circle without separation. All parts function as context for all other parts, each multi-functional in relationships, yet individually identifiable. Nothing has been constructed; nothing added or taken away. These relationships are inherent to the circle and are given form by the movement of touching any two points and creasing one diameter; not difficult to do or understand.

The circle shows a comprehensive, self-organizing dynamic movement in space. When touching any two points accurately there is the appropriate alignment forming proportional interaction between parts that reveal many functional relationships to be discovered; even with parts not yet formed by the continuation of folding the circle.

It becomes problematic when each part is defined in isolation as a separate function. We are left with learning to construct relationships symbolically represented by images of concepts and abstracted ideas. We have trouble teaching the foundations of mathematics when we do not know the greater context, have forgotten origin, and being unclear about what is principle to the endless profusion of parts.

The diameter is multifunctional, as all parts are. It is a line of symmetry, a perpendicular bisector, a median, an altitude, axis to a spherical pattern of movement, the sixth edge of a tetrahedron pattern, codified it is the directional function of AB=BA, and it represents the number one when removed from the circle. the diameter reveals the ratio of one Whole to two parts (1:2).

For any one that wants to fold the circle in half and will take the time to observation and contemplate what is there that was not there before the circle was folded, there is much to be discovered. We do not usually look for relationships of movement; our education directs us to abstract marks on paper. We must learn to look closer for what we can not see, for patterns that have yet to be formed, for the intervals, into the space where movement occurs between the marks; the unseen rhythms of pattern shifting proportions in the regularity of measure. The connection between math and music is that the scores of each are not what they represent. What stimulates the mind and soul is the physical changes occurring in the space represented by the intervals between markings. It is within the intervals that music is made in both mind and heart. It is in the movement of unity found in the circle that reveals functional interactions that mathematics describes.

Classroom experience

What does this look like in practice in a first and second grade classroom? Young children will explore and talk about their observations without needing to be right or wrong in what they describe. The limitation of their language sometimes makes for a more direct expression of what they see, but often less clear. In the primary grades students have not yet fully acquired the language of cultural logic through which they interpret and express their observations. It matters little what first catches the child’s attention, in what order, or manner of description, since everything they observe is interrelated and consistent to the folded circle.

First have them discuss the difference between symbols and real objects; the image of the sun and the experience of the sun, the difference between a circle and a ball, an image of each? How is a sphere different than all other forms? Where do these things come from? Cutting a spherical object in half will show two parts, two circles. We can continue cutting a part getting more pieces, but the object has been destroyed and unity lost. If we compressed the sphere, squished it flat, like flattening a ball of clay to a flat circle disk, it will change the form without destroying spherical unity. Nothing is added or taken away; the unity of the whole is still there; simply reformed to three circles in the form of a disk.

Draw a circle on paper. Cut the image from the paper, or use pre-cut circles. By playing with different ways the circle moves it can be discovered that when two opposite sides of the edge are touched together it makes a hole, a thin odd looking donut that is squeezed in at one point. In rolling the touching points on the edge it changes from a cylinder pattern to a cone pattern. The solid forms are not there, but there is enough to see that the cylinder has one surface parallel to itself in a circle and the cone shows two open circle ends of different sizes where the surface is no longer parallel. Nothing of the unity of the circle has changed except it has been moved in relationship to itself and again becomes reformed. The two further-most points on the edge forming the cylinder forms an open circle plane. A relationship is seen between the diameter of one circle as it becomes the circumference to another. This can all be talked about in common language. Math terms can be introduced into the discussion for clarity, economy, and connecting new ideas.

Folding

The next thing we might notice is that no matter where the two points are touching on the edge, when folded and creased will always divide the circle in six halves. Count the front, the back, and side to confirm this. When touching the two most opposite points on the circle and creasing will form a square relationship; four equally spaced points on the circumference. The crease divides the distance, at a right angle, between the two points of the unseen diameter. The one question to students is; “What do you see that was not there before?” From this one crease there are many relationships and associations that students can observe and discover for themselves by talking about what they see.

How the question is asked will often direct students in what to look for. If triangles are the lesson for the day then the teacher might ask what kind of shapes they see, or it can be left open to see if shapes are the first things they do see. It all leads back to how they folded the circle. Putting two imaginary points together and creasing generates two points and a creased line. With a new circle mark two points anywhere on the edge, touch the marked points together and crease. The two imaginary points are now visible using two marks. Now ask the question about what shapes they see. If none, then ask them to draw lines connecting all four points. (If appropriate, we can introduce a side discussion about points and lines.) Have them count all the lines on the circle (six cords). Then ask the question about shapes again. Not only will they see the shapes, they will have the experience of where they came from and that nothing was added that was not there form the very first fold of two imaginary points. They were just unable to see what was there until the relationships were given form by marking the points and tracing the distance between them. This helps in looking for unformed relationships as well as those formed.

What kind of shapes, how many, what sizes? How many different ways can they be combined? We can observe and describe isosceles, scalene, right and left hand right triangles; they are all interrelated in the same context, parts one to the others. We can see altitudes to the triangles, perpendicular bisectors, medians, etc. It is all there when you start looking for the multiple relationships between those five points and six cords. When the circle is lifted from the flat plane there are now five cords and one unseen variable, the straight line relationship between the two touching points. We see the space get smaller as the circle is folded to a flat semi-circle position. This proves that all half circles of the same size are congruent, containing the full circle. What happened in the folding? The movement pattern of a semi-sphere was created without leaving a form. If we removed the circle from the table and turned the edge all the way around, allowing the diameter to function as an axis it would make a spherical pattern without leaving a trace of a spherical form. At the same time a tetrahedron pattern of movement is reveled by the four points in space and the six relationships between them. (The solid form of the tetrahedron comes eight creases later in reforming the circle.)

When asked what else they see in the spherical movement, a student will see an inside, suggesting to another an outside. When moving the circle in both directions we see the two insides and outsides change places; they are reciprocal to each other. One is the inverse to the other and can be talked about as a reflective movement of symmetry by folding the circle into halves. We can call one direction positive and the other negative. With two perpendicular bisectors to the kite we can then divide the radii into positive and negative directions out from the center point of crossing. When the circle is out flat there are two sets of right hand and left hand triangles. When the circle is folded shut the triangles are congruent, going in the same direction, no longer opposites of right and left, positive and negative. By saying one side is positive and the other side negative we can then see, one positive and one negative tetrahedron. There are many questions to be asked in guiding students to go deeper into their observations. It is not beyond five and six year old students to talk about what they see, and to introduce mathematical terms to clarify their observations. With older students this can serve as a good review of what they should know and be able to identify by connecting abstract book information to what they have folded. You find out exactly what students know by what observations they make, how they express what they see, and the connections they are finding.

Numbers

Looking at *one* circle, understanding there are *two* sides and the circle ring between the two shows *three* parts to the circle, (1,2,3). This is reflected in folding the circle where the Whole (O). when creased (1) forms two parts (2). One line and two parts can not be separated making three (3). Joining the total of three parts with the two previous individualized parts we have a total of five parts (5). This gives us the number of points, four points on the circumference and point of intersection when open flat. This counting process shows in a number progression, relationships from the undifferentiated Whole to points, to a folded line, and to areas. It is the points that define relationships of lines that show the areas. The number sequence looks like this (O,1,2,3,5) The next number is pretty simple; add three to the five, we get eight. Eight is the number of individual areas in this fold. You don't even have to know numbers to do it. This is not an adding one at a time sequence of numbers. We have added one to two getting three and the two to three getting five and three to five getting eight. That can be endlessly repeated in an orderly progression of one step backwards and two steps forward.

See how far a class can go with this sequencing. This is fundamental number progression and grows from the undifferentiated Whole of the circle/sphere. Then by adding one number at a time, or by two and threes, multiplying by sets and other kinds of groupings, many kinds of progressive sequencing can be discovered and played with.

This number sequence called the Fibonacci series, for the man who first discovered it, should not take away from students the opportunity to discover this for themselves. When many of the "higher" math concepts and functions are presented within the circle, where all parts are connected and interrelated, it is easier for students to understand from their own experiential observations, discussed in their own words.

Adding numbers that describe the properties of the tetrahedron pattern; four circumference points and six relationships, we get the number ten. Adding all the numbers of the properties of the tetrahedron four points, four planes, and the six edge relationships is number fourteen. By adding the one and four we get five, the primary number quality of the tetrahedron corresponding to what we have just counted. Now there are two tetrahedra in the movement of the circle. So two fives make ten, which is simple the diameter removed from the circle to form the symbol (1O), which represents the number count of four points and six edges of the tetrahedron pattern. Numbers are an easy way to see diverse connections between patterned relationships in different forms.

Conclusion

Folding circles along with drawing the image allows students a deeper and more comprehensive understanding of what it is to really observe what they are seeing, to discern the difference between pattern, form and design, to think deeply about something, to problem solve through observation in the largest possible context. It makes "higher level" math and geometry concepts easily accessible to young students giving them an experiential and progressive grounding. Nothing is necessary beyond a paper circle and the students own observations. Within the self organizing of the circle they now have a meaningful place to understand all the bits and pieces of fragmented curriculum information. They have experienced the multifunctional and interrelated aspect of parts where a change with one will always affect relationships to all other parts. There is a systematic organizational geometry inherent within the circle: nothing is arbitrary or random about what is generated. Students will have an understanding of the Wholeness of the circle that is principled to the patterns and arrangements of all other parts. Mathematics is a language used to formulize generalizations about combinations of relationships and associations of parts within the circle. This is only the beginning of folding circles. More complex mathematical functions are revealed in the continuation of folding, reforming, and combining in multiples, circle/sphere unity.

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